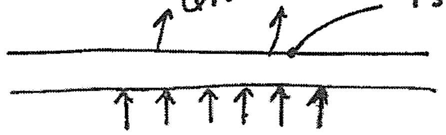
1. $T_\infty = 20°C$

$\dot{Q}_{rad}$ $\dot{Q}_{conv}$ $T_s = 275°C = 548K$



$$\dot{Q}_{loss} = \dot{Q}_{rad} + \dot{Q}_{conv}$$

$$=$$

$$\dot{Q}_{rad} = \cancel{\varepsilon}^{1} A \sigma T_s^4 = A_s 5.67 \cdot 10^{-8} \frac{W}{m^2K^4} \cdot (548K)^4$$

$$\dot{Q}_{rad} = 5113 \ W/m^2 \cdot \frac{\pi D^2}{4} = 15.7 \ W$$

$$\dot{Q}_{conv} = \text{natural convection from flat plate} = \bar{h}A(T_s - T_\infty)$$

$\bar{h}$ from Nusselt # (table 9-1)

$$L = A_s/P = \frac{\pi D^{\cancel{2}}}{4 \cancel{\pi D}} = \frac{D}{4}$$

$$= \frac{25 \ cm}{4} = 0.0625 \ m$$

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha}$$

fluid (air) properties at $T_{film} = 147.5°C = 420.5 \ K$

$k = 0.034 \ W/mK$

$$\beta = \frac{1}{T} = 0.00238 \ 1/K \quad (B)$$

$$\nu = 2.831 \cdot 10^{-6} \ m^2/s \quad (N)$$

$$\alpha = 4.026 \cdot 10^{-5} \ m^2/s \quad (A)$$

$$Ra = \frac{9.8 \ m/s^2 \cdot 0.00238 \frac{1}{K} (255K)(0.0625m)^3}{2.831 \cdot 10^{-6} \ m^2/s \cdot 4.026 \cdot 10^{-5} \ m^2/s}$$

$$= 1.27 \cdot 10^8, \text{ so}$$

$$Nu_L = 0.15 \ Ra_L^{1/3}$$

$$= 75$$

$$\bar{h} = \frac{Nu \cdot k}{L} = 41.1 \ W/m^2K$$

$$\dot{Q}_{conv} = 41.1 \frac{W}{m^2K} \cdot \frac{\pi \cancel{D^2}}{4} \cdot \underbrace{(T_s - T_\infty)}_{255K}, \quad D^2 \to (.0625m)^2$$

$$\dot{Q}_{conv} = 32 \ W$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{netrad}$$

$$= 32 \ W + 15.7 \ W$$

$$= 47.7 \ W$$

2.

a. To what minimum temperature must a filament be heated so 10% of energy is visible ($\lambda < 0.8 \mu m$)

assume: black or grey body so f_λ (blackbody fcns) apply.

for $\lambda T > 2200 \, \mu m K$, $f_\lambda > 0.10$ (10%)

if $\lambda < 0.8 \mu m \rightarrow T \geq \frac{2200 \, \mu m K}{0.8 \, \mu m} = \boxed{2750 K}$

b. at what wavelength will emission be maximum?

Wien's law $\lambda_{max} T = 2898 \, \mu m K$

$$\lambda_{max} = \frac{2898 \, \mu m K}{2750 K} = \boxed{1.053 \, \mu m}$$

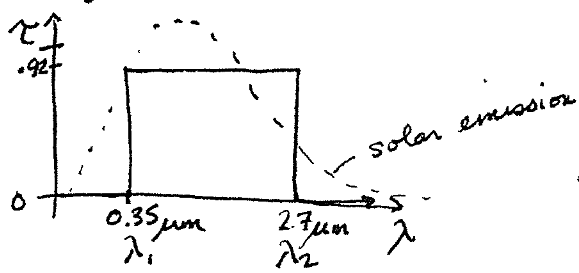
$\downarrow$

$\boxed{\text{Infrared}}$

max emission as heat, not visible light!

3. Glass transmits 92% of radiation in λ range 0.35–2.7 μm. It is opaque otherwise. What % of solar radiation will it transmit?

given τ



assume sun is bb radiator at 5800K.

$\lambda_1 T_{sun} = .35 \cdot 5800 = 2030 \, \mu m K$

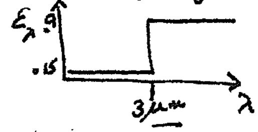
$\lambda_2 T_{sun} = 2.7 \cdot 5800 = 15660 \, \mu m K$

step 1: what fraction of sunlight is between 0.35 & 2.7 μm?

$f_{\lambda_1 T - \lambda_2 T} = f_{0-\lambda_2 T} - f_{0-\lambda_1 T} = 0.97 - 0.07 = 0.90$

step 2: 92% of this light is transmitted: % transmitted = $\tau \cdot f_{.35-2.7} = .92 \cdot .9 = .83$ or 83%

4. The emissivity of a surface is 0.15 for $\lambda < 3\mu m$ & 0.9 for $\lambda > 3\mu m$



a) what is $\bar{\varepsilon}$ for $T_{surface} = 5800\ K$?

$$\bar{\varepsilon} = \varepsilon_{0\to 3\mu m} \cdot f_{0-3\mu m} + \varepsilon_{3\mu m \to \infty} f_{3\mu m \to \infty}$$

$$= 0.15 \cdot f_{0-3\mu m} + 0.9 \cdot (1 - f_{0-3\mu m})$$

λT for $\lambda = 3\mu m$, $T = 5800\ K \rightarrow \lambda T = 17400 \mu m\ K$

$f_{17400} = 0.979$

$$\bar{\varepsilon}_{5800K} = 0.15(0.979) + 0.9 \cdot (0.021)$$

$$\boxed{\bar{\varepsilon}_{5800K} = 0.166.}$$

b) same for $\bar{\varepsilon}_{300K} \Rightarrow$ $\lambda T = (3\mu m \cdot)(300\ K) = 900 \mu m K$

$f_{900\mu m K} = 0.000169$

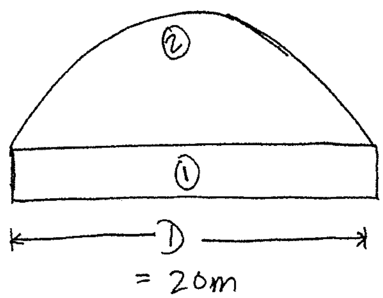
$$\bar{\varepsilon}_{300K} = 0.15 \cdot (0.000169) + .9(.999832)$$

$$\boxed{\bar{\varepsilon}_{300K} = 0.900}$$

c) the absorptivity of this surface to sunlight is equal to its emissivity for a spectral profile at 5800K so $\boxed{\alpha_{solar} = 0.166}$

d) this surface is a better solar reflector. It only absorbs 16% of energy at solar wavelengths, but it emits 90% of possible energy at room temp.

5.



a) $\cancel{F_{1\to1}}^{\,0} + F_{1\to2} = 1$

$F_{2\to1} + F_{2\to2} = 1$

$F_{2\to2} = 1 - F_{2\to1}$

$A_2 F_{2\to2} = A_2(1 - F_{2\to1})$

$A_2 F_{2\to2} = A_2 - A_2 F_{2\to1}$

$A_2 F_{2\to2} = A_2 - A_1 F_{1\to2}$

$$F_{2\to2} = \frac{A_2 - A_1 F_{1\to2}}{A_2}$$

$A_1 = \pi D^2/4 = 314 m^2$

$A_2 = \frac{1}{2} 4\pi (D/2)^2 = 628 m^2$

$$F_{2\to2} = 1 - \frac{A_1}{A_2} = 1 - \frac{314}{628}$$

$$\boxed{F_{2\to2} = \frac{1}{2}}$$

b) $\dot{Q}_{net,2\to1} = A_2 \sigma F_{2\to1}(T_2^4 - T_1^4)$

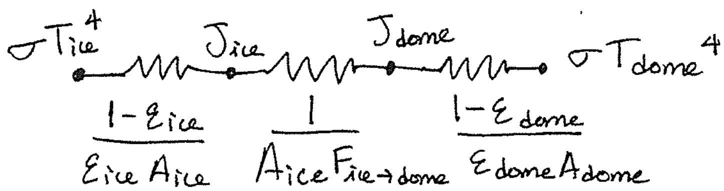
$= A_1 F_{1\to2} \sigma (T_2^4 - T_1^4)$

$= (314 m^2)(5.67\times10^{-8} W/m^2K^4)(288 K^4 - 268 K^4)$

$$\boxed{\dot{Q}_{net,2\to1} = 30.7 kW}$$

c) $\varepsilon_{ice} = 0.9$

$\varepsilon_{dome} = 0.1$



$$\dot{Q}_{dome \to ice} = \frac{\sigma \left(T_{dome}^4 - T_{ice}^4\right)}{R_{total}}$$

$$R_{total} = \frac{1-\varepsilon_{ice}}{\varepsilon_{ice} A_{ice}} + \frac{1}{A_{ice} F_{ice \to dome}} + \frac{1-\varepsilon_{dome}}{\varepsilon_{dome} A_{dome}}$$

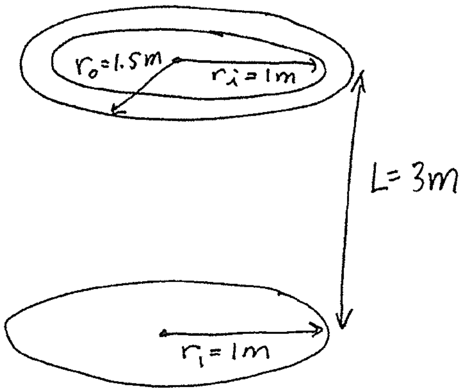
$$= \frac{1-0.9}{0.9(314)} + \frac{1}{(314)(1)} + \frac{1-0.1}{0.1(628)}$$

$$= 0.0179$$

$$\dot{Q}_{dome \to ice} = \frac{(5.67 \times 10^{-8})(288^4 - 268^4)}{0.0179}$$

$$\boxed{\dot{Q}_{dome \to ice} = 5.46 \text{ kW}}$$

6.



$$F_{disk \to rng} = F_{disk \to r_o} - F_{disk \to r_i}$$

$$F_{disk \to r_o} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{r_o}{r_1}\right)^2 \right]^{1/2} \right\}$$

$$S = 1 + \frac{1 + R_o^2}{R_1^2}, \quad R_o = r_o/L, \quad R_1 = r_1/L$$

$$F_{disk \to r_i} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2}, \quad R = \frac{r_i}{L} = \frac{r_1}{L}$$

$$F_{disk \to rng} = \frac{1}{2}\left\{ 1 + \frac{1 + \left(\frac{r_o}{L}\right)^2}{\left(\frac{r_1}{L}\right)^2} - \left[\left(1 + \frac{1 + \left(\frac{r_o}{L}\right)^2}{\left(\frac{r_1}{L}\right)^2} \right)^2 - 4\left(\frac{r_o}{r_1}\right)^2 \right]^{1/2} \right\}$$

$$- \left\{ 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \right\}$$